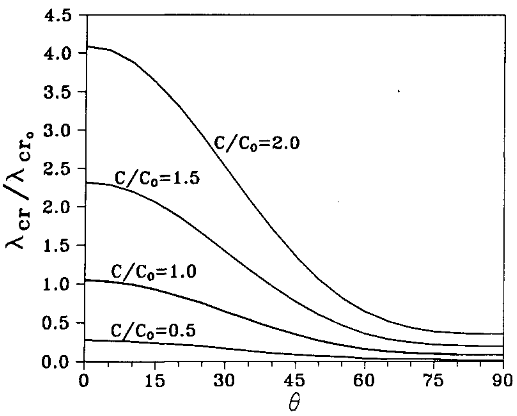

Fig. 3 Effects of fiber orientation on flutter boundary.

coalesce to Z_{cr} at a value of $\lambda = \lambda_{cr}$, which is a critical value of dynamic pressure, and become a complex conjugate pair for $\lambda > \lambda_{cr}$. The solution procedures used here represent an exact solution rather than a modal solution of the differential equation and hence do not possess convergence difficulties.

Numerical Results

The panel considered for the analysis is a $[0/\pm 45/90/\mathrm{Core}]_{\mathrm{sym}}$ sandwich plate. AS4/3501-6 graphite epoxy is used for the face sheets and Al honeycomb is used for the core. The material constants are $E_2 = 10{,}130$ N/mm^2, $E_1 = 11.45E_2$, $G_{12} = 0.543E_2$, $\nu_{12} = 0.3$ for the face sheets, and $G_{xz} = 0.0429E_2$ for the core. The thickness of each lamina is 0.125 mm. The core thickness is set to be 10 mm. Figure 2 shows the effects of thickness and transverse shear modulus of the core on the flutter boundary of the panels with four different stacking sequencies. All flutter boundaries have been normalized to $\lambda_{cr_0} = 155.9$ of the $[0/\pm 45/90/\mathrm{Core}]_{\mathrm{sym}}$ panel with $c_0 = 10$ mm and $G_{xz_0} = 0.0429E_2$. The solid lines represent cases with $G_{xz} = G_{xz_0}$ and dashed lines represent cases with $c = c_0$. It shows that G_{xz} has negligible effect on the flutter boundaries since changing G_{xz} with fixed c will not affect the bending stiffness of the plate which governs the flutter boundary. But changing c with fixed G_{xz} will have pronounced effect on the flutter boundary since the bending stiffness of the plate is mainly dependent on the core thickness. Furthermore, with the same number of lamina and fiber orientation, the stacking sequence of the face sheets has small effect on the flutter boundary. Figure 3 shows the effect of fiber orientation on the flutter boundary of $[\pm\theta/\mathrm{Core}]_{\mathrm{sym}}$ sandwich panels (λ_{cr} is also normalized to λ_{cr_0} of the $[0/\pm 45/90/\mathrm{Core}]_{\mathrm{sym}}$ panel). For all different core thickness, the highest flutter boundary is obtained with the fiber aligned with the x axis; rotating the fiber away from the x axis results in a continuous reduction in flutter boundary for values of θ up to 90 deg.

Conclusions

A flutter motion equation for a two-dimensional composite sandwich panel is derived by considering the total lateral displacement as the sum of the displacement due to bending of the plate and that due to shear deformation of the core. The aerodynamic theory is based on the piston theory with first-order approximation. The Mach number considered is limited to beyond approximately 1.6. This derivation can be extended to the general composite sandwich panel system for practical application in the aeronautical industry. The results show that the composite sandwich panel greatly improves the flutter boundary over the corresponding composite laminated panel if it has a proper core thickness. The results also show that the transverse shear G_{xz} of the core has negligible effects on flutter boundaries.

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Criterion for Decoupling Dynamic Equations of Motion of Linear Gyroscopic Systems

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I. Introduction

THE damping matrix of a general linear structural vibration system is a very important physical parameter in describing the differential equations of motion of the system. In analyzing the dynamic responses of such a system, the damping term should never be neglected. In the case of the general damping matrix, which consists of conventional damping and gyroscopic damping and can be expressed in terms of symmetrical and nonsymmetrical matrices,[1] the system equations of motion can seldom be decoupled in their corresponding real mode space. Therefore, complex modal theories are conventionally employed to perform a dynamic analysis of the system. This makes the analyses and computations much more complicated than necessary. In this Note, a system with general damping is changed to an undamped one using a transformation matrix of function in general coordinates. The obtained undamped system is not equivalent to that of a decoupled one in the real mode space. The necessary and sufficient condition for this transformation is derived. As a consequence, the Caughey's condition, which is necessary and sufficient to decouple the dynamic equations of motion of a symmetrical linear damped system in terms of the classical real modes, is extended to nonsymmetrical systems.

II. Basic Equations

For a linear structural vibration system with a general form of nonsymmetric damping matrix denoted by $\ll M, C, K \gg$, its differential equations of motion take the following form:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{f(t)\} \tag{1}$$

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If $[M]$ is nonsingular, Eq. (1) can be reduced to

$$\{\ddot{x}\} + 2[A]\{\dot{x}\} + [B]\{x\} = [M]^{-1}\{f(t)\} \tag{2}$$

where

$$[A] = \tfrac{1}{2}[M]^{-1}[C] \tag{3}$$

and

$$[B] = [M]^{-1}[K] \tag{4}$$

Substituting

$$\{x\} = [V(t)]\{q\} \tag{5}$$

gives

$$[V(t)]\{\ddot{q}\} + 2([\dot{V}(t)] + [A][V(t)])\{\dot{q}\} + ([\ddot{V}(t)] + 2[A][\dot{V}(t)] + [B][V(t)])\{q\} = [M]^{-1}\{f(t)\} \tag{6}$$

where $[V(t)]$ is a matrix function that makes the coefficient matrix of the time derivative of the generalized coordinate $\{q\}$ zero. Thus,

$$[\dot{V}(t)] + [A][V(t)] = 0 \tag{7}$$

where V is the Jacobi matrix.

One of the elementary solutions to this equation is

$$[V(t)] = \exp(-[A]t) \tag{8}$$

Because $\exp(-[A]t) = \exp([A]t)^{-1}$ is true for an arbitrary square matrix A, Eq. (6) can be rearranged to give

$$\{\ddot{q}\} + [D]\{q\} = [M]^{-1}\{F(t)\} \tag{9}$$

where

$$[D] = \exp([A]t)[B - A^2]\exp(-[A]t) \tag{10}$$

$$\{F(t)\} = [M]\exp([A]t)[M]^{-1}\{f(t)\} \tag{11}$$

and $[D]$ is a matrix function of time. Thus, generally, the system defined by Eq. (9) is a time-varying system, which does not offer real natural frequencies and classical modes. Therefore, conventional real modal methods of analysis cannot be employed in dynamic response analyses of such systems.

III. Conditions of Changing to the Equivalent Undamped System

Only when $[D]$ is independent of time does the equivalent undamped system governed by Eq. (9) have real natural frequencies and classical normal modes. The necessary and sufficient condition for $[D]$ to be a constant matrix is that its differential matrix, namely $[D_t']$, is zero with respect to time. From Eq. (10), we arrive at

$$[D_t'] = \exp([A]t)([A][B] - [B][A])\exp(-[A]t) = 0 \tag{12}$$

Because $\exp([A]t)$ and $\exp(-[A]t)$ are nonsingular, this requires that

$$[A][B] - [B][A] = 0 \tag{13}$$

so that A and B are commutative. In this case, we get

$$[D] = [D]|_{t=0} = [B - A^2] \tag{14}$$

Substituting for $[A]$ and $[B]$ from Eqs. (3) and (4) and inserting the result in Eq. (9) leads to the equivalent undamped system becoming

$$[M]\{\ddot{q}\} + [K_{eq}]\{q\} = \{F(t)\} \tag{15}$$

where the equivalent stiffness matrix is

$$[K_{eq}] = [K] - \tfrac{1}{4}[C][M]^{-1}[C] \tag{16}$$

The excitation $\{F(t)\}$ is determined by Eq. (11) and the commutative condition becomes

$$[C][M]^{-1}[K] = [K][M]^{-1}[C] \tag{17}$$

Equation (17) is identical to what Caughey and O'Kelley[2] presented as a necessary and sufficient condition of decoupling a damping matrix in the real mode space of a symmetrical system $\ll M, K \gg$. Thus, according to Eq. (12), which leads to Eq. (13) and eventually Eq. (17), it is concluded that a damped system, whether symmetric or not, can be transformed into an equivalent undamped one, if and only if the Caughey's condition is satisfied. However, Eq. (17) cannot be simply used to determine if a nonsymmetrical general damping matrix can be decoupled.

IV. Decoupling a General Damping Matrix

Caughey's condition is true for any symmetrical system with a nonsingular mass matrix.[1] Rayleigh damping of both symmetrical and nonsymmetrical systems, for which

$$[C] = [M]\sum_{i=0}^{n}\alpha_i([M]^{-1}[K])^i \tag{18}$$

is a special case that can be decoupled in the real mode space of the system.

A simple counterexample is presented as follows:

$$[M] = \begin{bmatrix} m & 0 \\ 0 & 2m \end{bmatrix} \qquad [C] = \begin{bmatrix} 0 & -c_{21} \\ c_{21} & 0 \end{bmatrix} \qquad [K] = \begin{bmatrix} k & 0 \\ 0 & 2k \end{bmatrix}$$

Its $\ll M, K \gg$ has a repeated eigenvalue. The associated eigenvectors are

$$\begin{Bmatrix} 0 \\ 1 \end{Bmatrix}, \qquad \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$$

and any nonzero linear combinations of these two vectors, namely

$$\begin{Bmatrix} \alpha \\ \beta \end{Bmatrix}$$

Choose arbitrarily two linearly independent eigenvectors

$$\{V_1\} = \begin{Bmatrix} \alpha_1 \\ \beta_1 \end{Bmatrix} \qquad \text{and} \qquad \{V_2\} = \begin{Bmatrix} \alpha_2 \\ \beta_2 \end{Bmatrix}$$

To decouple the damping matrix $[C]$, there should be

$$\{V_1\}^T[C]\{V_2\} = 0$$

It follows

$$\alpha_1\beta_2 - \alpha_2\beta_1 = 0 \tag{19}$$

According to the operation rules of vectors, this relation indicates that the vector product of $\{V_1\}$ and $\{V_2\}$ is zero. So $\{V_1\}$ and $\{V_2\}$ share the same direction and, accordingly, are linearly dependent. Hence, the damping matrix cannot be decoupled by the real modes.

A more general damping matrix can be decoupled if and only if

$$(i) \qquad [C][M]^{-1}[K] = [K][M]^{-1}[C]$$

and if

$$(ii) \qquad [\bar{C}_L] = [Y_L^T][C][X_L]$$

is not defective[3] in the real domain. $[X_L]$ and $[Y_L]$ are the right and the left eigenvectors corresponding to an L times repeated eigenvalue. It is obviously true of the necessity. To prove the sufficiency, suppose that the right and the left real mode matrices are denoted by $[X]$, $[Y]$, respectively. Hence,

$$[M] = [X][Y]^T \tag{20}$$

$$[\bar{K}] = [Y]^T[K][X] = \text{diag}(\alpha_1, \alpha_2, \ldots, \alpha_n) \tag{21}$$

Note $[\bar{C}] = [Y]^T[C][X]$, then condition (i) changes into $[\bar{C}][\bar{K}] = [\bar{K}][\bar{C}]$. Therefore, elements on both sides must be equal, i.e., $\bar{c}_{ij}\alpha_i = \bar{c}_{ij}\alpha_j$. If α_i is not equal to α_j, $\alpha_i \neq \alpha_j$, we will get $\bar{c}_{ij} = 0$ $(i \neq j)$. If α_i is an L times repeated eigenvalue, according to condition (ii) that $[\bar{C}_{ij}]$ is not defective in the real domain, there exist $[U]$, $[V]$ (dimensioned $L \times L$) to make

$$[V]^T[\bar{C}_L][U] = \text{diag}(\beta_1, \beta_2, \ldots, \beta_L) \tag{22}$$

Since $[X_L]$ and $[Y_L]$ are the eigenvectors corresponding to the L times repeated eigenvalue α_i, linear combinations $[X_L][U]$, $[Y_L][V]$ are also the eigenvectors of α_i. Consequently, $[C]$ can be decoupled in the real mode space.

A simple criterion to see whether $[\bar{C}]$ is defective or not in the real domain is that if $\{e_x\}^T\{e_y\} = 0$, then $[\bar{C}]$ is defective, where $\{e_x\}$ and $\{e_y\}$ are the left and the right eigenvectors of the same eigenvalue, respectively. If $\ll M, K \gg$ gives no repeated eigenvalues, $L = 1$; or if $\ll M, C, K \gg$ is of symmetry, which results in the symmetry of $[\bar{C}]$ and nondefectiveness in the real domain, then condition (ii) is naturally satisfied in these two cases. Thus, Caughey's condition becomes the necessary and sufficient one.

But if Eq. (17) is met with, the system $\ll M, C, K \gg$ is inevitable to be transformed into an equivalent undamped system described by Eq. (15), which is not the same as the decoupled one from the original system.

When $[C]$ can be decoupled, $[K_{eq}]$ can be simultaneously decoupled. In this case, Eq. (15) is reduced to

$$\{\ddot{p}\} + \left[\diagdown\ \omega_i^2 - \tfrac{1}{4}\bar{c}_{ii}^2\ \diagdown\right]\{p\} = [Y]^T\{F(t)\} \tag{23}$$

V. Conclusions

On the basis of this analysis, a conclusion can be drawn that, if the Caughey's condition holds, a general $\ll M, C, K \gg$ system can be transformed into its equivalent undamped system so as to perform an analysis in the real domain instead of employing complex modal theories. Thus, the meaning of the Caughey's condition is broadened and its applicability is extended. In terms of decoupling general vibration equations, an extra condition is added in making such a determination.

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Nonlinear Free Vibration Characteristics of Laminated Anisotropic Thin Plates

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Introduction

LAMINATED composite plates and shell panels are being used in aerospace and other engineering applications as lightweight high-strength structural components. Analysis of free vibrations is one of the important design considerations for these components. The linear theory of vibrations predicts the frequency of natural vibrations to be independent of the amplitude. In many instances, if the amplitude of the vibration is large, such a result will not be justified due to one or another of the nonlinear effects. In general, the interest in vibrations of nonlinear systems centers on the vibrations of large amplitudes. Nonlinear vibrations of plates have received considerable attention in recent years. A comprehensive review of literature may be found in Refs. 1–4. The basic requirements for the analysis are the formulation of the equation of motion from the energy functions of the system and its solution for obtaining the vibration characteristics. The characteristic of the system, viz., the restoring force function in the equation of motion, is found to be a cubic polynomial, which is in the form of Duffing type or a combination of quadratic and cubic terms. The solution of the equation of motion (i.e., the nonlinear frequency of the plates that is a function of material properties, dimensions of the plates, and the amplitude of vibration) is obtained by several methods such as the perturbation method,[5–8] the harmonic balance method,[9] exact integration,[10–14] the iterative numerical schemes,[15,16] and the finite element method.[17] Some of the conventional tools for the analysis of nonlinear oscillations—such as the averaging techniques, multiple-time scaling, and harmonic balancing—are described in Ref. 18.

The usual perturbation techniques are inappropriate, if the coefficient of the nonlinear terms in the differential equation does not involve small parameters.[19] Mickens[20] has indicated that the only generally applicable technique in such a situation is the method of harmonic balance. Telban et al.[21] have recently demonstrated that the hybrid-Galerkin method improves the perturbation approximations of the Duffing equation. In recent years, Singh et al.[11] have shown that the perturbation solution suggested in Ref. 6 fails for the case of a simply supported two-layered antisymmetric cross-ply rectangular plate with immovable edges. The purpose of the present Note is to examine this case by solving the equation of motion through the hybrid-Galerkin method. The results are comparable with the exact solution of the equation of motion. The approximate solution of the equation of motion through the hybrid-Galerkin method discussed in this Note is quite simple and accurate and not only provides the frequency ratios for the specified amplitude ratios but also gives the expression for the displacement as a function of time.

Equation of Motion

The equation of motion for the nonlinear free vibration of a thin plate governed by the second-order nonlinear ordinary differential equation in time variable is of the form[5–16]

$$\frac{d^2W}{d\tau^2} + \omega_L^2 f(W) = 0 \tag{1}$$

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